ESTIMATION OF DISTANCE ERROR BY FUZZY SET THEORY REQUIRED FOR STRENGTH DETERMINATION OF HDR $^{192}$Ir BRACHYTHERAPY SOURCES

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Abstract

Verification of the strength of high dose rate (HDR) $^{192}$Ir brachytherapy sources on receipt from the vendor is an important component of institutional quality assurance programme. Either reference air-kerma rate (RAKR) or air-kerma strength is the recommended quantity to specify the strength of gamma emitting brachytherapy sources. The use of Farmer-type cylindrical ionization chamber of sensitive volume $0.6 \text{ cm}^3$ is one of the recommended methods for measuring RAKR of high dose rate (HDR) $^{192}$Ir brachytherapy sources. While using the cylindrical chamber method, it is required to determine the positioning error of the ionization chamber with respect to the source which is called the distance error. An attempt has been made to apply the fuzzy set theory to estimate the subjective uncertainty associated with the distance error. A simplified approach of applying this fuzzy set theory has been proposed in the quantification of uncertainty associated with the distance error. In order to express the uncertainty in the framework of fuzzy sets, the uncertainty index was estimated and was found to be within 2.5% which further indicates that the possibility of error in measuring such distance may be of this order. This indicates that the relative distance $l_i$ values estimated using analytical methods are within 2.5% uncertainty. This value of uncertainty in distance measurement should be incorporated in the uncertainty budget while estimating the expanded uncertainty in HDR $^{192}$Ir source strength measurement.

Introduction

Verifying the strength of high dose rate HDR $^{192}$Ir brachytherapy sources on receipt from the vendor is an important component of institutional quality assurance programme[1,2]. The recommended quantity to specify the strength of gamma emitting brachytherapy sources is either reference air-kerma rate (RAKR) or air-kerma strength (AKS). Calibration of the $^{192}$Ir sources used in HDR remote afterloading brachytherapy units is carried out either by using a thimble ionization chamber (in-air jig method) or by using a well-type ionization chamber. A Farmer-type cylindrical ionization chamber of nominal
sensitive volume of 0.6 cm³ is frequently used for in-air calibration of HDR $^{192}$Ir brachytherapy sources in addition to a suitable well-type ionization chamber as cylindrical ionization chambers are readily available in the hospitals[3-4].

A 370 GBq (10 Ci) $^{192}$Ir source provides an ionization current of only about $1 \times 10^{-11}$ amp in a 1.0 cm³ ionization chamber at a distance of 20 cm[5]. It is true that very near to a brachytherapy source, the radiation intensity changes very rapidly due to inverse-square law. A 0.1 cm error in a 10 cm distance causes a 2% error in calibration[6]. Small errors in positioning the chamber can translate into large errors in the estimation of source strength. Increasing the separation between centers of the chamber and the source will improve the measurement accuracy. However, this will result in proportionate reduction in the current leading to larger percentage contributions by leakage current and gamma-ray scattering from the room surroundings and poor reproducibility. Getting closer of course worsens the distance error and requires a large geometric correction for the size and shape of the ionization chamber[7].

The seven distance method is recommended as a standard method to maximize the accuracy in measuring the strength of HDR brachytherapy sources by using cylindrical ionization chamber[4-6]. While using the cylindrical chamber method, it is required to determine the positioning error of the ionization chamber with respect to the source which is commonly called as the distance error. Earlier, we have developed the analytical methods to estimate the distance error[8-9]. As further research in this work, an attempt has been made to apply the fuzzy set theory to estimate the subjective uncertainty associated with the distance error. Since the measured distance possesses some error during the measurement and the input components are imprecise, fuzzy set theory is an appropriate tool to estimate the uncertainty present in such an ambiguity[10].

Materials and Methods

Multiple distance measurement technique

The microSelectron-HDR unit from Nucletron was used in this work. This unit uses an old design micro-Selectron $^{192}$Ir HDR brachytherapy source with 370 GBq (10 Ci) nominal activity to treat brachytherapy patients with high dose rate comparable to teletherapy.

To determine experimentally, the RAKR of an HDR $^{192}$Ir brachytherapy source using a PTW 0.6 cm³ Farmer-type cylindrical ionization chamber, a multiple distance measurement technique was used. This measurement has historically been made at seven separate distances. Thus the technique has been termed the ‘7 distance’ measurement (7 DM). While using cylindrical ionization chamber for measurement of the strength of HDR $^{192}$Ir brachytherapy sources, it is necessary to estimate three items, viz. (i) the positioning error ($\pm$) of the ionization chamber with respect to the source, (ii) the contribution of scatter radiation ($M_s$) from the floor, walls, ceiling, and other material in the treatment room and (iii) a proportionality constant. The 7DM was suggested to determine these parameters and thereafter the strength of HDR $^{192}$Ir brachytherapy sources[4-5]. Kumar et al. [8-9] has described in detail the procedure for measuring the RAKR of HDR $^{192}$Ir brachytherapy sources. In this 7 DM method, the output of the source in air is measured at seven different distances each corresponding to a meter reading $M_d$, which is the sum of primary and scattered radiation

$$M_d = M_p + M_s \quad (1)$$

where $M_p$ is the meter reading due to primary radiation only and $M_s$ is the meter reading due to scattered
radiation only, which is assumed to be independent of distance. As the primary radiation follows inverse square law, Equation (1) can be written as

$$M_{p} = \left( M_{d} - M_{s} \right) = \frac{f}{(d + c)^{2}} \text{ where } i = 0 \ldots 6 \quad (2)$$

where \(d_{i}\) is the apparent distance between source and the chamber centers, ‘c’ is the offset error in the distance measurement and \(f\) is a proportionality constant which is independent of distance. On solving Equation (2), one may obtain the following functional form for relative distance \(l_{i}\) between the successive measurement points \((i = 1, 2, \ldots, 6)\)

$$l_{i} = (d_{i} - d_{i-1}) = \frac{f_{i}}{d_{i} + c} \left[ \frac{1}{M_{d_{i}}} \right] \left[ \frac{1}{M_{a_{i}}} \right] + \frac{M_{d_{i}} f_{i}}{2} \left[ \frac{1}{M_{d_{i}}} \right] \left[ \frac{1}{M_{a_{i}}} \right] \quad (3)$$

where \(M_{d_{i}}\) is the meter reading at distance \(d_{i}\) from the source. Equation (3) has two unknowns \((f, M_{s})\) which were determined by bi-variate linear regression analysis by adopting the least square method. After obtaining \(f\) and \(M_{s}\), the value of ‘c’ was determined using Equation (2). Having determined the value of \(f\), the air-kerma rate (AKR) (Gys-1) can be calculated using the formula

$$AKR = \frac{N_{K} f}{(d + c)^{2} \Delta t} \quad (4)$$

where, \(N_{K}\), is the interpolated air-kerma calibration coefficient of the chamber for HDR 192Ir brachytherapy source and \(\Delta t\) is the time interval of the measurement. RAKR can then be determined using the following equation

$$RAKR = AKR \left( \frac{d + c}{d_{ref}} \right)^{2} \quad (5)$$

where \(d_{ref}\) is the reference distance of 1 m.

**Fuzzy Set theory**

Zadeh introduced the fuzzy set as a class of object with a continuum of grades of membership[10-11]. In contrast to classical crisp sets, the fuzzy approach relates to a grade of membership between 0 and 1. The membership function of a fuzzy set, \(A\) is defined in the form of a triangular or trapezoidal fuzzy number as shown in Fig. 1(a-b). The details about fuzzy set may be found elsewhere[11]. An algorithm for implementing alpha cut representation of fuzzy set theory to compute the associated uncertainty is presented below

**Algorithm to compute \(\alpha\)-cut representation of distance**

1. Given a fuzzy parameter, say, constant of proportionality, \(f\) (see Equation (2)) as a triangular fuzzy number: \(<f> = <f_{LB}, f_{most\ likely}, f_{UB}> = <1945, 1972, 1999>\), we have the \(\alpha\)-cut representation as \([f_{\alpha}^{LB}, f_{\alpha}^{UB}] = [1950.4, 1993.6]_{\alpha=0.2}\).

2. In a similar way, \(\alpha\)-cut representations of all other fuzzy parameters are constructed.

3. \(\alpha\)-cut representation being an interval number, we use the interval arithmetic operation of

$$\left[ f \right]^{1/2} \times \left( \left[ \frac{1}{M_{d}} \right]^{1/2} - \left[ \frac{1}{M_{a}} \right]^{1/2} \right)$$

$$= \left[ f_{\alpha}^{LB}, f_{\alpha}^{UB} \right]^{1/2} \times \left( \left[ \frac{1}{M_{d_{\alpha}}} \right]^{1/2} - \left[ \frac{1}{M_{a_{\alpha}}} \right]^{1/2} \right) \times \left( \frac{1}{M_{d_{\alpha}}} \right) - \left( \frac{1}{M_{a_{\alpha}}} \right)$$

4. \([A, B] = [f_{\alpha}^{LB}, f_{\alpha}^{UB}]^{1/2} \times [\eta, \lambda] = \min \left\{ f_{\alpha}^{LB} \eta, (f_{\alpha}^{LB})^{1/2} \eta, (f_{\alpha}^{LB})^{1/2} \lambda, (f_{\alpha}^{LB})^{1/2} \lambda \right\} \times \left[ \frac{1}{M_{d_{\alpha}}} \right]^{1/2} - \left( \frac{1}{M_{a_{\alpha}}} \right) \right\}$$

where, \(\eta = \frac{1}{M_{d_{\alpha}}^{1/2}} - \left( \frac{1}{M_{d_{\alpha}}} \right) \lambda = \frac{1}{M_{a_{\alpha}}} - \left( \frac{1}{M_{a_{\alpha}}} \right) \lambda \)

5. \(\alpha\)-cut representation being an interval number, we use the interval arithmetic operation of
Finally, the alpha cut value of 0.5 of a fuzzy set was used to quote the bounds of the uncertainty of the imprecise or vague information applied to any physical quantity because the uncertainty bounds of the input triangular fuzzy parameters are taken as one sigma level, i.e., \( f_{\text{LB,UB}} = (f_{\text{most likely}} \pm \sigma) \) according to the principle of measurement uncertainty. Hence, \( l_i \) values were chosen for 0.5 alpha cut value [10] and compared with the analytically estimated values. Here, in this case, the bounds are positive numbers and hence in case of multiplication operation of two intervals, we have applied restricted Dong, Shah and Wang (DSW) algorithm[10].

Support of a triangular fuzzy number is defined as the range of the extremes at \( \alpha \)-cut = 0 as shown in Fig. 1(c). From Fig. 1(c), we can write the support of a fuzzy set as \( S = (R-P) \), where, \( R \) and \( P \) are the two extreme bounds. In order to express the uncertainty in the framework of fuzzy sets, we define uncertainty index[11] as the ratio of the support to the most likely value (crisp value at membership equal to 1). Again, from Fig. 1(c), uncertainty index of the given fuzzy set is written as \( U = (S/Q) \), where, \( Q \) is the most likely value. Here, in this paper, we have estimated the uncertainty index for each relative distance measured experimentally.

**Results and Discussion**

We have estimated the uncertainty of the relative distance, \( l_i = (d_i - d_0) \) for experimentally measured distances such as 5, 10, 15, 20, 25 and 30 cm. The membership function for one measured distance (say 10 cm) is shown in Fig. 2. It can be interpreted from Fig. 2 that the membership function of the distance \( l_i \) for measurement distance is turned out to be a triangular in shape because the initial consideration of the subjective based uncertain parameters are taken into consideration as “around the measured value”. Since the measurement uncertainty is always quoted at one sigma level, fuzzy set theory based approach of uncertainty quantification is also quoted at an equivalent level, and here this is alpha cut value of 0.5 of the fuzzy set. Results of \( \alpha \)-cut = 0.5 of the fuzzy distance \( l_i \) along with experimental and
analytical values of \( l_i \) are presented in Table 1 and it can be seen that analytical value as well as the experimentally measured relative distance lie within the bounds of the subjective uncertainty of the \( l_i \). Support of each membership values corresponding to each experimental distance and the associated uncertainty index are further shown in Table 2. It can be seen from Table 2 that the uncertainty indices remain invariant for all the experimentally measured distances indicating that each and every triangular fuzzy membership function is normalized and convex. Maximum value of the uncertainty index is found to be within 2.5% which further indicates that the possibility of error in measuring such distance may be of this order.

<table>
<thead>
<tr>
<th>Experimental</th>
<th>Analytical ( l_i )</th>
<th>Fuzzy set theory ( (l_i)_{L} )</th>
<th>( (l_i)_{U} )</th>
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<tbody>
<tr>
<td>5</td>
<td>4.999</td>
<td>4.968</td>
<td>5.031</td>
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<tr>
<td>10</td>
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<td>9.907</td>
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<tr>
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<td>24.840</td>
<td>25.119</td>
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<tr>
<td>30</td>
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<td>29.829</td>
<td>30.180</td>
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**Table 2: Support and uncertainty index of relative distance \( l_i \)**

<table>
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<tr>
<th>Experimental</th>
<th>Support</th>
<th>Uncertainty Index</th>
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<tbody>
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<td>0.025</td>
</tr>
<tr>
<td>10</td>
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<td>25</td>
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<td>0.022</td>
</tr>
<tr>
<td>30</td>
<td>0.702</td>
<td>0.023</td>
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**Conclusions**

Uncertainty of the positioning error, the so called "distance error", of the ionization chamber with respect to the source was evaluated. Fuzzy set theory was applied for this evaluation due to the subjectivity involved in the experimental facility. Uncertainty in the possible input parameters was addressed as triangular fuzzy number. Propagation of uncertainty of the input parameters is carried out on the basis of the model described in this work (see sub-section: algorithm to compute \( \alpha \)-cut distance representation of distance) via the alpha cut of a fuzzy set. The crisp values of \( l_i \) estimated using analytical method lie within the bounds computed using fuzzy set theory. This indicates that \( l_i \) values estimated using analytical methods are within 2.5% uncertainty. This value of uncertainty in distance measurement should be incorporated in the uncertainty budget while estimating the expanded uncertainty in HDR \(^{192}\text{Ir}\) source strength measurement.

**References**


